

BOARD OF TECHNICAL EXAMINATION – KARNATAKA

SUBJECT: APPLIED MATHEMATICS – I

For I- semester

DIPLOMA COURSES OF ALL BRANCHES

Contact hour per week: 04

Contact hour per Semester: 64

UNIT NO.	CHAPTER TITLE	CONTACT HR.
	ALGEBRA	
1	DETERMINANTS	2
2	MATRICES	4
3	BINOMIAL THEOREM	3
4	LOGARITHMS	2
5	VECTOR ALGEBRA	6
	TRIGONOMETRY	
6	UNITS AND MEASUREMENT OF AN ANGLE	3
7	TRIGONOMETRIC RATIOS OF ACUTE ANGLE AND STANDARD ANGLES	6
8	TRIGONOMETRIC RATIOS OF ALLIED ANGLES	4
9	HEIGHTS AND DISTANCES	2
10	COMPOUND ANGLES	4
11	MULTIPLE AND SUB MULTIPLE ANGLE	2
12	TRANSFORMATION FORMULAE	4
13	PROPERTIES AND SOLUTIONS OF TRIANGLES	4
14	INVERSE TRIGONOMETRIC FUNCTIONS	2
	ANALYTICAL GEOMETRY	
15	BASIC CONCEPTS OF ANALYTICAL GEOMETRY	6
16	STRAIGHT LINES	6
17	TESTS AND ASSIGNMENTS	4
	TOTAL HOUR	64

REFERENCE BOOKS:

1. Applied Mathematics -I By W.R Neelakanta Sapna Publications.
2. Applied Mathematics -I By Dr. D S Prakash S Chand Publications
3. Text book of mathematics for P U C

**QUESTION PAPER BLUE PRINT
FOR APPLIED MATHEMATICS – I**

UNIT NO.	HOUR	Questions to be set (2 Marks) PART- A	Questions to be set (5 Marks) PART- B	Questions to be set (5 Marks) PART- C	Questions to be set (5 Marks) PART- D
1	2	1	1	-	-
2	4	1	2	-	-
3	3	1	1	-	-
4	2	-	1	-	-
5	6	2	3	-	-
6	3	-	-	1	-
7	6	1	-	2	-
8	4	1	-	2	-
9	2	-	-	1	-
10	4	1	-	1	-
11	2	1	-	1	-
12	4	1	-	1	-
13	4	1	-	2	-
14	2	1	-	1	-
15	6	1	-	-	2
16	6	2	-	-	2
TOTAL	64	15	8	12	04
Questions to be answered		10	5	8	3

APPLIED – MATHEMATICS – I

	<u>Marks</u>	No. of Questions to be set	No. of Questions to be Answered
Part A	2 X 10 = 20	15	10
Part B	5 X 05 = 25	08	05
Part C	5 X 08 = 40	12	08
Part D	5 X 03 = 15	04	03

Total = 100

CONTENTS OF APPLIED MATHEMATICS – I

ALGEBRA

UNIT – 1: DETERMINANTS. 2 Hr.

Definition, Order, Expansion of 2nd & 3rd order Determinants by means of examples. Problems on finding unknown quantity in a 2nd & 3rd determinant using expansion. Solving simultaneous linear equations by determinant method (Cramer's rule). Problems.

UNIT – 2: MATRICES. 4 Hr.

Determinant value of a square matrix. Singular and non singular matrices with examples. Minor and co factor of an element of a matrix. Adjoint of a matrix. Problems. Characteristics equations of a square matrix and its roots. Problems. Cayley – Hamilton's theorem (statement only). Problems. Find inverse of a matrix using the theorem.

UNIT – 3: BINOMIAL THEOREM. 3 Hr.

Meaning of ${}^n C_r$ and its value. Binomial theorem for $(x + a)^n$, where n is a positive integer. Expansion. Finding constant term, co-efficient of x^n , particular term and middle term(s). Problems.

UNIT – 4: LOGARITHEMS. 2 Hr.

Definition of common and natural logarithms. Laws of logarithms. Problems on laws.

UNIT – 5: VECTOR ALGEBRA. 6 Hr.

Definition of vector. Representation of a vector as a directed line segment. Magnitude of a vector. Types of vectors. Position vector. Vector can be expressed in terms of end points of position vector. Addition and subtraction of vector in terms of line segment. Vector in a plane and vector in a space in terms of unit vector i, j and k respectively. Product of vectors. Scalar and vector product. Geometrical meaning of scalar and vector product. Applications of dot(scalar) and cross(vector) product: Projection of a vector on another vector. Area of parallelogram and area of triangle. Work done by a force and moment of force

TRIGONOMETRY

UNIT – 6: UNITS AND MEASUREMENT OF AN ANGLE

3 Hr

Defination of an angle. Systems of unit of an angle. Defination of radian. Radian is a constant angle. Relation between degree & radian and problems. Derivation of $l = r\theta$ & $A = \frac{1}{2} r^2\theta$. Problems.

UNIT –7: TRIGONOMETRIC RATIOS OF AN ACUTE ANGLE.

6 Hr

Defination of Trigonometric ratios in terms of sides of a triangle for an acute angle. Proof of Trigonometric identities. Trigonometric ratios of standard angles: The numerical value of trigonometric ratios of standard angles like 0^0 , 30^0 , 45^0 , 60^0 and 90^0 . Problems.

UNIT – 8: ALLIED ANGLES.

4 Hr.

Angle of any magnitude, sign of the trigonometric ratios. Meaning of allied angles. Trigonometric ratios of allied angles in terms of θ . Problems. Complimentary angles and relation between trigonometric ratios of complimentary angles. Problems.

UNIT – 9: HEIGHTS AND DISTANCES.

2 Hr.

Defination of angle of elevation and depression. Problems.

UNIT -10: COMPOUND ANGLES.

3 Hr.

Geometrical proof of $\sin(A + B)$ and $\cos(A + B)$. Find $\tan(A + B)$ using $\sin(A + B)$ and $\cos(A + B)$. Write the formulae for $\sin(A - B)$, $\cos(A - B)$ and $\tan(A - B)$. Problems.

UNIT -11: MULTIPLE ANGLES.

3 Hr.

Derive ratios of multiple angles of $2A$ and $3A$. Problems. Obtain sub multiple angle formulae. Problems.

UNIT-12: TRANSFORMATION FORMULAE.

4 Hr.

Express sum or difference of Sine and Cosine of an angles in to product form. Express product of Sine and Cosine of angles in to sum or difference form. Problems.

UNIT-13: PROPERTIES AND SOLUTION OF TRIANGLES.

4 Hr.

The relation between sides of a triangle and Sines, Cosines and Tangents of any angle(Sine rule, Cosine rule and Tangent rule), Projection rule. Half angle formulae in terms of sides of a triangle. Problems. Problems on solution of triangles (four types).

UNIT- 14: INVERSE TRIGONOMETRIC FUNCTIONS.

2 Hr.

Defination. Principle values of inverse trigonometric functions. Derivation of results like $\sin^{-1}x + \cos^{-1}x = \pi/2$ $\tan^{-1}x + \cot^{-1}x = \text{cosec}^{-1}x + \sec^{-1}x$, $\tan^{-1}x + \tan^{-1}y = \tan^{-1}$ Problems.

ANALYTICAL GEOMETRY

UNIT-15: BASIC CONCEPTS OF ANALYTICAL GEOMETRY.

6 Hr.

Defination of a point in a plane, Specification of a point using co-ordinate system. Points on X-axis and Y-axis. Derivation of distance formula. Problems. Section formulae. Derivation of co-ordinate of a point which divide the line internally in the given ratio(No derivation for external division). Mid point formula. Problems. Centroid, area of a triangle and collinear points. Problems. Locus of a point with respect to a fixed point and with respect to two fixed points and its equations. Problems.

UNIT-16: STRAIGHT LINES.

6 Hr.

Inclination of a line with horizontal line and its slope. Intercept of a straight line. Slope of a line parallel to X-axis and Y-axis. Derivation of conditions for two lines to be parallel and perpendicular. Problems. Derivations equations of straight lines $y = mx + c$, $y - y_1 = m (x - x_1)$, $y - y_1 = (y_2 - y_1 / x_2 - x_1) (x - x_1)$, $(x / a) + (y / b) = 1$ and $x \cos\alpha + y \sin\alpha = P$. General equation of a line $ax + by + c = 0$ and problems. Equation of lines through a point and parallel or perpendicular to a given line. Problems. Angle between two lines. Problems. Point of intersection of lines. Equation of a line through the point of intersection which is parallel or perpendicular to the given line. Problems.

GENERAL AND SPECIFIC OBJECTIVES

ALGEBRA

UNIT – 1: DETERMINANTS.

2 Hr

GENERAL OBJECTIVES.

1.1 To understand meaning of determinants and evaluation of 2nd & 3rd order determinants.

1.2 To understand the solution of linear equations by Cramer's rule.

SPECIFIC OBJECTIVES.

1.1.1 Definition of Determinant by means of Algebraic Expression and order in terms of Rows and Columns.

1.1.2 Evaluate Determinants of 2nd & 3rd order --- **Problems.**

1.1.3 Solve the problems to find unknown quantity in a determinant.

1.2.1 Solve the linear equations of 2nd or 3rd unknowns using determinants. **Problems**

UNIT – 2: MATRICES.

4 Hr

GENERAL OBJECTIVES.

2.1 To understand Minors and co-factors, adjoint and inverse of a matrix.

2.2 To know the characteristic equation for 2 x 2 matrix.

SPECIFIC OBJECTIVES.

2.1.1 Define singular and non singular square matrix.

2.1.2 Find the minors and cofactors of each elements of matrix

2.1.3 Define adjoint of a square matrix.--- **Problems.**

2.1.4 Define inverse of a square matrix --- **Problems.**

2.2.1 Explain characteristic equation of a 2 x 2 matrix and its roots --- **Problems.**

2.2.2 State Cayley – Hamilton's theorem for 2nd order (Statement only) --- **Problems.**

2.2.3 Apply it to find inverse of the matrix --- **Problems.**

UNIT – 3: BINOMIAL THEOREM.

3 Hr

GENERAL OBJECTIVES.

- 3.1 To recognise the meaning of ${}^n C_r$ & to compute the values ${}^n C_0, {}^n C_n, {}^n C_1, {}^n C_2, \dots, C_r$**
3.2 To prove ${}^n C_r = {}^n C_{n-r}$ by demonstration.
3.3 To know the statement of $(x+a)^n$ for positive integral Index n.

SPECIFIC OBJECTIVES.

- 3.1.1 Meaning of ${}^n C_r$ and value of ${}^n C_r$.
 3.2.1 Use of ${}^n C_r$ in the expansion $(x + a)^n$, where n is positive integer.
 3.3.1 Expansion of $(x + a)^n$, where n is positive integer. (only statement).
 3.3.2 Solve the problems of the following types
 (a) expansion of type $(2x + y)^6, (\sqrt{3} + 2)^4, (x^2 - 1/\sqrt{x})^8$
 (b) To simplify $(\sqrt{2} + 1)^5 + (\sqrt{2} - 1)^5$
 (c) To find middle term(s)
 (d). To find the coefficient of x^n and constant terms in $(x+a)^n$

UNIT – 4: LOGARITHMS

2 Hr

GENERAL OBJECTIVES.

- 4.1 To understand logarithm, common logarithm and natural logarithm.
 4.2 To comprehend laws of logarithm.

SPECIFIC OBJECTIVES.

- 4.1.1 Define logarithm, common logarithm and natural logarithm.
 4.2.1 Derive the laws of logarithm
 (a) $\log_a mn = \log_a m + \log_a n$
 (b) $\log_a m/n = \log_a m - \log_a n$
 (c) $\log_a x^k = k \log_a x$
 (d) $\log_a b = \log_c b / \log_c a$ (CHANGE OF BASE)
 4.2.2 Solve problems on laws of logarithms

UNIT – 5: VECTOR ALGEBRA

6 Hr

GENERAL OBJECTIVES.

- 5.1 To understand vectors.**
5.2 To understand vector operations.

SPECIFIC OBJECTIVES.

- 5.1.1 Define a vector and represent a vector geometrically.
 5.1.2 Name a directed line segment as a vector.
 5.1.3 Write different types of vectors.

- 5.1.4 Write addition and subtraction of vectors in terms of the segment.
- 5.1.5 Explain the position vector of a point with reference to a point.
- 5.1.6 Write the conclusion of difference of two vectors with same initial point as position vector of a point.
- 5.1.7 Define vector in a plane and vector in a space in terms of unit vectors i, j and k respectively.
- 5.2.1 Write types of vector product (a) Scalar product (b) Vector product
- 5.2.2 Define scalar product and vector product.
- 5.2.3 Interpret product of vectors geometrically.
- 5.2.4 Design the product of vectors to find angle between them and unit vector
- 5.2.5 Apply product of vectors to find (i.) projection of a vector on another vector
(ii) Area of triangle and parallelogram (iii) work done by a Force (application of scalar product) (iv) moment of a force (application of vector product)

TRIGONOMETRY

UNIT – 6: UNIT AND MEASUREMENT OF ANGLE

3 Hr

GENERAL OBJECTIVES.

6.1 To understand the concept of angle, unit and its measurement.

SPECIFIC OBJECTIVES.

- 6.1.1 Define an angle.
- 6.1.2 Define a right angle in different systems.
- 6.1.3 Define radian.
- 6.1.4 Obtain relations between radian and degree.
- 6.1.5 Solve problems.
- 6.1.6 Prove the radian as a constant angle.
- 6.1.7 Derive $l = r\theta$
- 6.1.8 Derive area of sector = $(r^2\theta) / 2$.
- 6.1.9 Solve related problems.

UNIT – 7: TRIGONOMETRIC RATIOS

4 Hr

GENERAL OBJECTIVES.

- 7.1 To understand trigonometric ratios.**
- 7.2 To comprehend trigonometric ratios of standard angles.**

SPECIFIC OBJECTIVES.

- 7.1.1 Define trigonometric ratios for an acute angle.
- 7.1.2 Derive important formulae (identities)
- 7.1.3 Solve problems using identities.

- 7.1.4 Obtain trigonometric ratios of standard angles 0° , 30° , 45° , 60° , 90°
7.1.5 Solve problems using standard values.

UNIT – 8: ALLIED ANGLES

4 Hr

GENERAL OBJECTIVES.

8.1 To understand allied angles of the type ($n\pi/2 \pm \theta$).

SPECIFIC OBJECTIVES.

- 8.1.1 Define positive and negative angles.
8.1.2 Give examples to differentiate positive and negative angles.
8.1.3 Explain trigonometric ratios in different quadrants and assign signs.
8.1.4 Define allied angles in different quadrants.
8.1.5 Write formulae of $90 \pm \theta$, $180 \pm \theta$, $270 \pm \theta$, $360 \pm \theta$ and $(-\theta)$
8.1.6 Solve problems on allied angles.

UNIT - 9: HEIGHTS AND DISTANCES

2 Hr

GENERAL OBJECTIVES.

9.1 To understand the measurement of Heights and Distances using trigonometry.

SPECIFIC OBJECTIVES.

- 9.1.1 Define angle of elevation and angle of depression.
9.1.2 Define the “Height” and “Distance”
9.1.3 Solve problems on height and distance.

UNIT – 10: COMPOUND ANGLES

3 Hr

GENERAL OBJECTIVES.

10.1 To understand compound angles.

SPECIFIC OBJECTIVES.

- 10.1.1 Define a compound angle.
10.1.2 Give examples for compound angles.
10.1.3 Derive the formula of $\sin(A+B)$ and $\cos(A+B)$ geometrically.
10.1.4 Obtain $\tan(A+B)$ in terms of $\tan A$ and $\tan B$ using $\sin(A+B)$ and $\cos(A+B)$.
10.1.5 Obtain formula for $\sin(A-B)$, $\cos(A-B)$ and $\tan(A-B)$.

10.1.6 Solve the problems on compound angles.

UNIT – 11: MULTIPLE AND SUBMULTIPLE ANGLES.

3 Hr

GENERAL OBJECTIVES.

11.1 To understand multiple and sub multiple angles.

SPECIFIC OBJECTIVES.

11.1.1 Define multiple and submultiple angle with example.

11.1.2 Derive the formulae for trigonometric ratios of $2A$ and $3A$ using compound angle formulae.

11.1.3 Deduce half angle formulae from the ratio of $2A$ formulae.

11.1.4 Solve problems.

UNIT – 12: TRANSFORMATION FORMULAE

4 Hr

GENERAL OBJECTIVES.

12.1 To understand transformation formulae.

SPECIFIC OBJECTIVES.

12.1.1 Obtain expressions for $\sin C \pm \sin D$ and $\cos C \pm \cos D$ in terms of Product of trigonometric ratios.

12.1.2 Derive expressions for $\sin A \cos B$, $\cos A \sin B$, $\cos A \cos B$ and $\sin A \sin B$ in terms of the sum and difference of trigonometric ratios.

12.1.3 Solve problems.

UNIT – 13: PROPERTIES AND SOLUTIONS OF TRIANGLES

4Hr

GENERAL OBJECTIVES.

13.1 To understand the properties of triangles in order to solve them.

SPECIFIC OBJECTIVES.

13.1.1 Obtain sine rule, cosine rule and tangent rule for any angle of a triangle.

13.1.2 Write half angle formulae in terms of sides of a triangle (no proof).

13.1.3 Solve simple problems on above rules.

- 13.1.4 Find the solution of a triangle in the following cases when (i) all sides are given (ii) two sides and included angle is given (iii) two angles and one side is given. (iv) two sides and non included angle is given.

UNIT – 14: INVERSE TRIGNOMETRIC FUNCTIONS.

2 Hr

GENERAL OBJECTIVES.

14.1 To understand inverse trigonometric functions.

SPECIFIC OBJECTIVES.

14.1.1 Define inverse trigonometric function.

14.1.2 Write principal values of trigonometric functions.

14.1.3 Prove standard results like

$$1. \sin^{-1} x + \cos^{-1} x = \tan^{-1} x + \cot^{-1} x = \operatorname{cosec}^{-1} x + \sec^{-1} x = \pi/2,$$

$$2. \tan^{-1} x \pm \tan^{-1} y = \tan^{-1} \left(\frac{x \pm y}{1 \mp xy} \right)$$

14.1.4 Solve the problems on inverse functions.

ANALYTICAL GEOMETRY

UNIT – 15: BASIC CONCEPTS OF POINT AND LOCUS

6 Hr

GENERAL OBJECTIVES.

15.1 To understand the basic concepts of point and locus.

SPECIFIC OBJECTIVES.

15.1.1 Define coordinates in two dimensional space.

15.1.2 Define a point in terms of coordinates.

15.1.3 Obtain equation for the distance between two points in terms of co ordinates.

15.1.4 Solve problems on above formula.

15.1.5 Explain the section of a straight line.

15.1.6 Derive section formula for internal division.

15.1.7 Write section formula for external division.

15.1.8 Write formula for mid point of a line.

15.1.9 Solve problems on internal and external division.

15.1.10 Define locus of a point.

15.1.11 Solve problems on locus of a point.

UNIT – 16: STRAIGHT LINES

6 Hr

GENERAL OBJECTIVES.

16.1 To understand straight lines.

SPECIFIC OBJECTIVES.

16.1.1 Define angle of inclination.

16.1.2 Define slope of a straight line.

16.1.3 Define x and y intercepts for straight line.

16.1.4 Write $\tan \theta = \left(\frac{m_2 - m_1}{1 + m_2 m_1} \right)$

16.1.5 Write angle between two straight lines in terms of $\tan \theta = \left(\frac{m_2 - m_1}{1 + m_2 m_1} \right)$

16.1.6 Write the conditions for two lines to be parallel and to be perpendicular to each other.

16.1.7 Solve problems.

16.1.8 Derive the slope-intercept form of an equation of a straight line $y = mx + c$

16.1.9 Obtain the point – slope form equation to the straight line $y - y_1 = m(x - x_1)$

16.1.10 Obtain two point form of the equation to a straight line $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$

16.1.11 Obtain intercepts form of the equation to a straight line, $\frac{x}{a} + \frac{y}{b} = 1$

16.1.12 Derive the normal form of equation to a straight line, $x \cos \alpha + y \sin \alpha = p$

16.1.13 Write the general form of equation to a straight line, $ax + by + c = 0$

16.1.14 Solve problems on different forms of equation.

16.1.15 Solve problems to find an equation of a straight line passing through the given point and parallel to the given line.

16.1.16 Solve problems to find an equation of a straight line passing through the given point and perpendicular to the given line.

16.1.17 Solve problems to find an equation of straight line passing through the intersection of given lines.

16.1.18 Obtain the condition for the intersection of three lines express in general in the determinant form.

16.1.19 Solve the problems on the above condition.

REFERENCE BOOKS:

1. First and Second PUC mathematics Text Books of different authors.
2. Text Book of mathematics –I for polytechnic-by different authors
3. Text Book of mathematics-II for polytechnic- by different authors

BOARD OF TECHNICAL EXAMINATION – KARNATAKA

MODEL QUESTION PAPER

Code:

APPLIED MATHEMATICS –I (FOR ALL COURSES)

Time: 3 Hrs

Max.m marks:100

- NOTE:** i) Answer any 10 questions in section A, 5 questions in section B
8 questions in section C, & 3 questions in D
ii) Each question carries 2 marks in section A
ii) Each question carries 5 marks in remaining section

SECTION – A

1. Find x if $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & x \\ 7 & 8 & 9 \end{vmatrix} = 0$

2. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}$, Find A^{-1} adj(A)

3. Find the 5th term in $\left[\sqrt{x} + \frac{3}{\sqrt{x}} \right]^8$

4. If $\vec{a} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\vec{b} = \mathbf{i} + 2\mathbf{k}$, find $\vec{a} \cdot \vec{b}$

5. If $\vec{p} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$, Find unit vector in the direction of \vec{p}

6. If $\sin\theta + \operatorname{cosec}\theta = \frac{1}{2}$, find $\sin^2\theta + \operatorname{cosec}^2\theta$

7. If $\sec\theta = \operatorname{cosec}3\theta$, find θ .

8. Find the value of $\tan 15^\circ$ without using table or calculator.

9. Show that $\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$

10. Show that $\frac{\cos 17^\circ - \sin 17^\circ}{\cos 17^\circ + \sin 17^\circ} = \tan 28^\circ$

11. Show that $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

12. Show that $\cos^{-1}(-x) = \pi - \cos^{-1} x$

13. Find the locus of a point which moves at a equal distances from the axis

14. Find the inclination and Y-intercept of the line $\sqrt{3}x + y = 3$

15. Find the angle between the lines who's slopes are $\sqrt{3}$ and $1/\sqrt{3}$

SECTION – B

1. Solve the following equations by cramer's rule

$$2x + y = 1, y + 2z = 7 \text{ and } 3z - 2x = 11$$

2. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & 3 \\ 3 & 1 & 2 \end{bmatrix}$

3. Verify Caylay-Hamilton Theorem for the matrix $A = \begin{bmatrix} 2 & -1 \\ 1 & 5 \end{bmatrix}$

4. Find the co-efficient of x^{18} in $\left(x^2 + \frac{3a}{x}\right)^{15}$

5. If $\log_a(bc) = x$, $\log_b(ac) = y$ and $\log_c(ab) = z$. Show that $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = 1$

6. Find the sin of the angle between the vectors (3,2,-1) and (2,-1,4)

7. If the position vectors of A, B,C are respectively $i + 2j + 3k$, $2i + 5j - k$ and $-i + j + 2k$, find the area of triangle ABC

8. Find the projection of vector $2i + 3j + 2k$ on $4i + 5j + 3k$

SECTION – C

